W1-2-60-1-6

JOMO KENYATTA UNIVERSITY

OF

AGRICULTURE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS 2016/2017

THIRD YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF

BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE , FINANCIAL ENGINEERING, STATISTICS, AND BIOSTATISTICS

STA 2302: PROBABILITY AND STATISTICS IV

DATE: DECEMBER 2016 TIME: 2 HOURS

INSTRUCTIONS: Answer question ONE and any other TWO questions .

QUESTION ONE (30 MARKS)

(a) Define a multivariate data, giving an example.

[2 marks]

(b) Give an expression for a non-singular multivariate normal distribution and state the conditions such a distribution must satisfy to be a p.d.f

[4 marks]

(c) Define the following terms

(i) Probability generating function of a random vector

(ii) Central limit theorem with respect to a random vector

[4 marks]

(d) Consider a random vector $\underline{X}^T = [X_1, X_2, \ldots\ldots\ldots, X_7]$. The distribution of X_i is bernoulli with parameter p. Determine the distribution of $Y = \sum_{i=1}^{7} X_i$.

(e) Consider a random vector $\underline{X}^T = [X_1, X_2, X_3\ X_4]$ has covariance matrix .

$$\Sigma = \begin{bmatrix} 5 & 1 & 1 & 2 \\ 1 & 4 & 1 & 2 \\ 1 & 1 & 3 & -1 \\ 2 & 2 & -1 & 2 \end{bmatrix}$$

Find the covariance matrix of the random vector $\underline{Y}^T = [Y_1, Y_2, Y_3, Y_4]$ where $Y_1 = 2X_1 + X_2 - X_3 + X_4$, $Y_2 = X_1 + X_2 + X_3 + 2X_4$, $Y_3 = X_1 + X_3 - 3X_4$ and

$Y_4 = 3X_1 - X_2 + X_3 - X_4$

[4 marks]

(f) $\underline{X}$ is a p-variate normal random vector with mean $\underline{\mu}$ and covarince matrix Σ. Let the random vector $\underline{Y} = A^T\underline{X}$ where A^T is a $q \times p$ matrix of constants. Use characteristic function to show that

$\underline{Y} \sim N(A^T\underline{\mu}, A^T\Sigma A)$.

[3 marks]

(g) Suppose that $\underline{X}^T = [X_1, X_2, ..., X_p]$ are i.i.d random variable with a continuous distribution given by

$f(\underline{X}) = \left(\frac{1}{2\pi}\right)^{\frac{p}{2}} exp\left\{-\frac{1}{2}\sum_{i=1}^{p} X_i^2\right\}, \quad -\infty \leq X_i \leq \infty.$

Find the m.g.f of $\underline{X}$.

[7 marks]

QUESTION TWO (20 MARKS)

(a) Consider a 3-variate random vector $\underline{X}$ with joint probability density

$$f(\underline{x}) = \begin{cases} 6e^{-(x_1+x_2+x_3)}, & x_3 > x_2 > x_1 > 0 \\ 0 & elsewhere \end{cases}$$

Find the moment generating function of $\underline{X}$.

[6 marks]

(b) The following are loss amounts in thousand dollars from three portfolio of insurance policies;

Portfolio X_1:	19	29	30	34
Portfolio X_2:	23	26	29	41
Portfolio X_3:	21	27	33	39

Determine the portfolio's

(i) Mean vector

(ii) Variance Covariance matrix

(iii) Correlation matrix

[14 marks]

QUESTION THREE (20 MARKS)

(a) Let $\underline{X}^T = [X_1, X_2, ..., X_p]$ be normally distributed with mean $\underline{\mu}$ and covariance matrix Σ. Find the conditional distribution of $\underline{X}_1$ given that $\underline{X}_2 = \underline{x}_2$ where $\underline{X}^T = (\underline{X}_1^T, \underline{X}_2^T)$, $\underline{X}_1$ is a $q \times 1$ matrix and $\underline{X}_2$ is a $p - q \times 1$ matrix.

[12 marks]

(b) $\underline{X}^T = [X_1, X_2, X_3, X_4]$ is normally distributed with mean vector $\underline{\mu}^T = [3, 6, 1, 7]$ and covariance matrix

$$\Sigma = \begin{bmatrix} 11 & 5 & 2 & 3 \\ 5 & 4 & 1 & 0 \\ 2 & 1 & 3 & 0 \\ 3 & 0 & 0 & 2 \end{bmatrix}.$$

Find the parameters of the distribution of $\underline{X}^T = \left[X_1, X_3 / \underline{X}_2 = \begin{matrix} x_2 = 1 \\ x_4 = 4 \end{matrix} \right]$.

[8 marks]

QUESTION FOUR (20 MARKS)

The random vector $\underline{X}^T = [X_1, X_2, X_3]$ have a multivariate normal density given by

$f(\underline{x}) = C.exp\left\{\frac{-1}{2}Q\right\}$ where $Q = \frac{1}{17}(11x_1^2 + 7x_2^2 + 5x_3^2 - 6x_1x_2 - 4x_1x_3 - 2x_2x_3 + 2x_1 - 16x_2 - 22x_3 + 48)$

Determine:

(i) $\underline{\mu}$ and Σ

(ii) The constant C

(iii) The marginal density of X_1, X_3

(iv) The conditional distribution of X_1 given $X_2 = 2$, $X_3 = 2$.

[20 marks]