JOMO KENYATTA UNIVERSITY OF AGRICULTURE AND TECHNOLOGY

University Examinations 2016/2017

THIRD YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCES, BACHELOR OF SCIENCE IN BIOSTATISTICS, BACHELOR OF SCIENCE IN FINANCIAL ENGINEERING AND BACHELOR OF SCIENCE IN STATISTICS

STA 2311: STATISTICAL PROGRAMMING II

DATE: DECEMBER 2016   TIME: 2 HOURS

INSTRUCTIONS:
ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) Consider two matrices A and B. Write a function in R that accepts two matrices A and B and checks whether both A and B are square matrices and nonsingular. If the two matrices are not square and singular, the function returns the string "INVERSE DOES NOT EXIST" [4 marks]

b) The lifetime (X) of a certain type of a tire is normal in distribution with an average of three years and a standard deviation of 0.3 years. Using Monte carlo simulation and loop techniques, write an R program that computes $E(X^4)$ [6 marks]

c) In this survey, the response variable is called incidence; a value of 1 means that an island was occupied by a particular species of bird, and 0 means that the birds did not breed there. The explanatory variables are the area of the island Km^2 and the isolation of the island (distance from the mainland, km). The following output was obtained after the data was analyzed using R software.

Coefficients:

	Estimate	Std. Error	z value	Pr(>|z|)	
(Intercept)	6.6417	2.9218	2.273	0.02302	*
area	0.5807	0.2478	2.344	0.01909	*
isolation	-1.3719	0.4762	-2.877	0.00401	**

Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 68.029 on 49 degrees of freedom
Residual deviance: 28.402 on 47 degrees of freedom

i. Identify the type of regression fitted on this data. Explain.   [2 marks]

ii. Write down the fitted model.   [2 marks]

iii. Comment on the significance of the regression coefficients at 5% level of significance. Give an interpretation of these coefficients.   [4 marks]

d) Consider the following equation:

$$x^2 - 6x + 34 = 0$$

i solve for x   [2 marks]

ii write a function in R for evaluating the value of x   [2 marks]

e) Akaike Information Criterion is used is used to select the best model among competing models. Data was collected on 100 houses recently sold in Nairobi city. It consists of the sales prices(in $)$y$, house size (square feet) x_1, the number of bedrooms x_2 and the number of bathrooms x_3. Consider the following R output below(next page), use it to select the fitting model among the fitted models. Explain how you arrived at your answer.   [4 marks]

```
> mod1=lm(Y~X1)
> mod2=lm(Y~X2)
> mod3=lm(Y~X3)
> mod4=lm(Y~X1+X2)
> mod5=lm(Y~X1+X3)
> mod6=lm(Y~X2+X3)
> mod7=lm(Y~X1+X2+X3)
> AIC(mod1)
[1] 117.4994
> AIC(mod2)
[1] 117.021
> AIC(mod3)
[1] 114.9916
> AIC(mod4)
[1] 118.5212
> AIC(mod5)
[1] 116.9866
> AIC(mod6)
[1] 116.8284
> AIC(mod7)
[1] 118.8277
```

f) state and explain the assumptions made when fitting linear regression models.   [4 marks]

QUESTION TWO (20 MARKS)

a) One of the applications of eigen values and eigen vectors is in diagonalization of a matrix. Diagonalization means transforming a non diagonal matrix into equivalent matrix which is easier to deal with. Let $\underline{B}$ be a matrix with distinct eigen values and P be a matrix whose columns are the eigen vectors of the eigen values of B. Then, the product $P^{-1}BP$ is a diagonal matrix. Let

$$B = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 2 & -2 & 3 \end{pmatrix}$$

Write a program in R that does the following:

   i) Computes the diagonalizing matrix of B as per the instructions above.   [5 marks]

   ii) Computes the eigen values of B   [4 marks]

   iii) Verifies that the results in (i) above is a diagonal matrix with the results in (ii) as its diagonal elements   [6 marks]

b) An asset has a present value of Ksh. 5,000,000 which depreciates exponentially. The value of the asset at the end of X years is given as

$$Y = 5000000e^{-0.2(x^2+x+1)}$$

write a well commented program in R that computes:

   i. The value of the asset after 7.5 months   [4 marks]

   ii. The time it takes for the asset to depreciate by a third of its initial value   [2 marks]

QUESTION THREE (20 MARKS)

a) The elasticity of demand E is the percentage rate of decrease of demand per percentage increase in price. We obtain it from the demand equation according to the following formula:

$$E = -\frac{dq}{dp}\cdot\frac{p}{q}$$

where the demand equation expresses demand, q as a function of unit price p. To find the unit price that maximizes revenue, we express E as a functio of p, set E=1, and then solve for p. A factory has the demand equation

$$q = -\frac{5}{3000}P^2 + P + 1 \qquad (1)$$

   i) Write a program in R that computes $\frac{dq}{dp}$ from equation (1) above   [3 marks]

   ii) Given that

$$E = \frac{-\left(\frac{-10}{3000}P + 1\right)p}{\frac{-5}{3000}P^2 + P + 1} \qquad (2)$$

   write a program in R that computes the unit price that maximizes revenue   [4 marks]

iii) We say that the demand is elastic if $E > 1$, the demand is inelastic if $E < 1$ and the demand has unit elasticity if $E = 1$. Write a function in R that takes in the unit price p determined in a(ii) above to classify the E for this factory into either elastic, inelastic or unit elastic demand. [4 marks]

b) The temperature $T(in^0C)$ recorded during a day followed the curve

$$T = 0.001t^4 - 0.280t^2 + 25 \qquad (3)$$

where t is the number of hours from noon

[2 marks]

i) State the limits of t.

ii) Write a program in R that uses the *while* loop and Monte carlo simulation to determine the average temperature during a day under equation (3). [7 marks]

QUESTION FOUR (20 MARKS)

a) A system analyst studied the effect of computer programming experience on ability to complete within a specified time a complex programming task. Fifteen people were selected for the study. they had varying amounts of programming experience(in months). All people were given the same programming task, and the results of their success was recorded. The results was coded in binary fashion: $y = 1$ if the task was completed in the alloted time, and $y = 0$ if the task was not completed successfully.

months of experience(X)	5	7	8	10	18	19	9	9	17	19	5	7	8	10	18
Task success(Y)	1	0	0	1	0	1	1	1	0	0	1	0	0	1	0

i) What regression model would you fit to meet the objective of the study? Give reason(s) for your answer. [3 marks]

ii) Write a well commented R code code to read the data and fit the model in (i) above. [7 marks]

b) A company manufactures two products, X and Y by using three machines A,B and C. Machine A has 4 hours of capacity available during the coming week. Similarly, the available capacity of machines B and C during the coming week is 24 hours and 35 hours respectively. One unit of product X requires one hour of Machine A, 3 hours of machine B and 10 hours of machine C. Similarly one unit of product Y requires 1 hour, 8 hours and 7 hours of machine A,B ad C respectively. When one unit of X is sold in the market,it yields a profit of Kshs. 50/- per product and that of Y is Ksh. 70/- per unit.

[4 marks]

i) Formulate the linear program.

ii) write a well commented program in R that will be used to solve the above program. [6 marks]