W1-2-60-1-6

JOMO KENYATTA UNIVERSITY OF AGRICULTURE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS 2016/2017

FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN BIOSTATISTICS, STATISTICS, ACTURIAL SCIENCE AND FINANCIAL ENGINEERING.

STA 2204: CALCULUS FOR STATISTICS III

DATE: DECEMBER 2017 TIME: 2HOURS

INSTRUCTIONS: Answer question one (compulsory) any other two questions

QUESTION ONE (30 MARKS)

a)

i. Write the first three terms of the sequence $a_n = 5 - \frac{1}{n} + \frac{1}{n^2}$ (3 marks)

ii. Determine the convergence and the divergence of the sequence whose nth term is $a_n = \frac{n-1}{n} - \frac{n}{n-1}$ (3marks)

b) Obtain the McLaurin's series for $y(x) = \ln(x+2)$, up to and including the fourth term, use your series to estimate ln 2.4 to 6 decimal places. (5 marks)

c) Evaluate the integral correct to 6 decimal places $\int_0^4 \int_0^2 \frac{1}{(x+1)(y+1)} dydx$ (4 marks)

d) Six percent of electronic calculators manufactured by fx-series Company are defective. It can be shown that the probability of getting at least one defective plug in a random sample of n calculators is $f(n) = 1-(0.94)^n$. Consider the sequence $\{a_n\}$ defined by $a_n = f(n)$

i. Write down the terms a_{10}, a_{25} and a_{100} correct to 2 decimal places. (3 marks)

ii. Evaluate $\lim_{n \to \infty}(a_n)$ and interpret your results. (2 marks)

e) Find the radius and interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(x-2)^n}{n5^n}$ (4 marks)

f) Use the integral test to investigate the convergence and divergence of the series $\sum_{n=1}^{\infty} \frac{1}{(n+1)n}$ (3 marks)

g) Use the method of Lagrange multipliers to find the relative maximum of the function $f(x,y) = -2x^2 - y^2$ subject to the constraint $3x + 4y = 12$ (4 marks)

QUESTION TWO (20 MARKS)

a) The total monthly profit of Robertson controls company in manufacturing and selling x hundreds of its deluxe electronic setback thermostats and y mechanical thermostats per month is given by the total profit function $P(x) = -\frac{1}{8}x^2 - \frac{1}{2}y^2 - \frac{1}{4}xy + 13x + 40y - 280$ where P is in hundreds of dollars. If the production of setback thermostats is to be restricted to a total of exactly 4000 per month, how many of each model should Robertson manufacture in order to maximize its monthly profit? What is the maximum monthly profit? (9 marks)

b) Let R be the parallelogram bounded by the lines $x+y=1, x+y=2$ and $2x-3y=2$, $2x-3y=5$. Substitute $u=x+y, v=2x-3y$ to find its area $A=\iint_R dxdy$ (11 marks)

QUESTION THREE (20 MARKS)

a) Find the reduction formula for $I_n = \int \cos^n x dx \quad n \geq 2$ hence use your result to evaluate

$$\int_0^{\frac{\pi}{6}} \cos^5 x dx$$

(7 marks)

b) Evaluate the improper integral $\int_{-\infty}^{0} (1-x)e^{-x} dx$ (5 marks)

c) Determine whether the series $\sum_{n=0}^{\infty} \frac{n^n}{n!}$ converge or diverge? (4 marks)

d) Find the third Taylor polynomial for $f(x) = \sin x$ expanded about $a = \frac{\pi}{6}$ (4 marks)

QUESTION FOUR (20 MARKS)

a) By changing the order of integration, Evaluate $\int_0^2 \int_{\frac{y}{2}}^1 ye^{x^3} dxdy$ (7 marks)

b) The total weekly revenue (in dollars) that Acrosonic company realizes in producing and selling its book shell loudspeaker systems given by

$$R(x,y) = \frac{-x^2}{4} - \frac{3y^2}{8} - \frac{xy}{4} + 300x + 240y$$

Where x denotes the number of fully assembled units and y denotes the number of kits produced and sold per week. The total weekly cost attributable to these loudspeaker is $C(x,y) = 180x + 140y + 5000$ dollars where x and y as the same meaning as before. Determine how many assembled units and how many kits Acrosonic should produce per week to maximize its profit. (7 marks)

c) Solve the system $\begin{matrix} u = x + 2y^2 \\ v = x - 2y^2 \end{matrix}$ for x and y in terms of u and v. Then find the value of the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$ (6 marks)