

STATISTICS AND ACTUARIAL SCIENCE DEPARTMENT

CAT I

SMA 2231 DIFFERENTIAL EQUATIONS

DATE: 8/06/2018

TIME: ONE HOUR

INSTRUCTIONS: ANSWER ALL QUESTIONS

QUESTION ONE (15 MARKS)

- a) Explain each of the following
- i. Ordinary Differential equation (1 mark)
 - ii. Primitive (1 mark)

Find a differential equation associated with $y = c_1 e^x + c_2 e^{-2x} + c_3 e^{2x}$ (3 marks)

- b) The rate at which the population $x(t)$ of a certain species of insects (In hundreds) varies with time t (In weeks) is given by $\frac{dx}{dt} = 4t - t^2$ if there were 200 insects initially find the insects population at any time t (3 marks)

- c) Solve each of the following initial value problems

i. $x \frac{dy}{dx} - 2y = 3x^2, y(1) = 0.5$

ii. $(2x - 5y)dx + (4x - y)dy = 0, y(1) = 4$

QUESTION TWO (15 MARKS)

- a) The annual sales of a company are expected to grow at a rate proportional to the difference between the sales and an upper limit of K£5 millions. If the sales are 0 initially and K£1 million during the fourth year of operation. When will the sales be K£4 million (7 marks)

- b) Solve each of the following equations

i. $(e^v + 1) \cos u du + e^v (\sin u + 1) dv = 0$ (4 marks)

ii. $\frac{dy}{dx} + \frac{y}{x} = \frac{x}{y^3}$ (4 marks)

Handwritten notes for question 1b:

$$\int dx = \int (4t - t^2) dt$$

$$x = 2t^2 - \frac{1}{3}t^3 + C$$

At $t=0, x=200 \Rightarrow C=200$

$$x = 2t^2 - \frac{1}{3}t^3 + 200$$

Handwritten note for question 2bii:

$$\frac{dy}{dx} = 2x - 5$$

Handwritten notes for question 2bii:

$$(x^2 - 2.5y^2) dx + (2x^2 - \frac{1}{2}y^2) dy = 0$$

Handwritten notes for question 2bii:

$$B-C = A (x^2 - 2.5y^2) dx + 2x^2 - \frac{1}{2}y^2 = A$$

Handwritten note for question 2bii:

$$y(1) = 4$$

Handwritten note for question 2bii:

$$C u^2 - 2.5 v^2 + 2 C u^2 =$$